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Modeling and Controller Design for Quadruple Tank System

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***Abstract*- This paper presents PI controller Design for Process Control System. The new general form of plant model is developed for illustrating the highly flexible plant structure which can be adjusted to many styles that are used for the propose of giving control system engineers experience with multivariable control system design. This paper described about structure and physical properties of Modified Quadruple-Tank Process, Mathematical plant modeling, Analysis of plant transfer matrix characteristic and PI controller design.**

Keywords*- *Process Control System, Modeling, PI controller

I. INTRODUCTION

Most of industrial control problems are nonlinear and have multiple controlled variables that are common properties for the models of industrial processes to have significant uncertainties, strong interactions, and non-minimum phase behavior so it is important for control system engineer, chemical engineer to understand the non-idealities of industrial processes by carrying out experiments with a good laboratory apparatus [1]. A Modified quadruple tank process was designed and constructed to give control system engineers laboratory experience with key multivariable control concepts. The general form of plant model creating can keep the all properties of existing quadruple tank about multivariable zero locations and their directions of transfer matrix G which have intuitive physical interpretations in terms of how the valves γ_1 and γ_2 are set [2].

One of the main problems with mathematical models of physical systems is that the parameters used in the models cannot be determined with absolute accuracy. Inaccurate parameters can arise from many different factors. The values of parameters may change with time or various effects. These differences existing between the actual system and system model is called uncertainty [3][4].

However, the actual system parameters may change during operation or the input signal takes too large. In these cases, the linear model is no longer representing the actual system and causes practical problems. Therefore, a robust controller is needed to stabilize these types of systems for the entire range of expected variations in the plant parameters.

The remainder of this paper is organized as follows: Section 2 discusses the details of modified quadruple tank system. In Section 3, the model development is done. Section 4 represents the transfer function matrix of the system. Section 5 is the further analysis of the system. In Section 6 represents the structure of PI controller design. Section 7 shows simulation results followed by conclusion in Section 8.

II. MODIFIED QUADRUPLE TANK PROCESS

The modified quadruple tank process is a combination of two double tank system is shown in Fig.1.

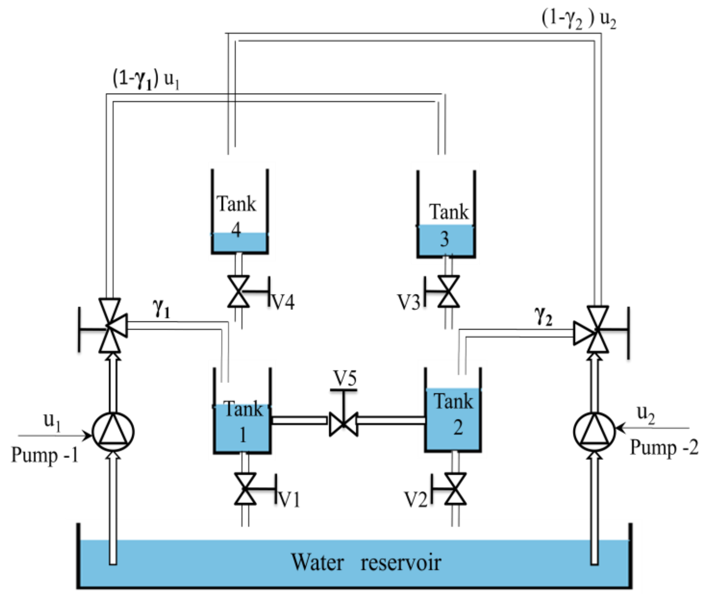

Fig.1. Schematic diagram of Modified Quadruple Tank Process System

This setup consist of a water supply tank with two variable speed positive displacement pump (capacity 0-200V, 1ph) for water circulation fitted with flow dampers, four transparent process tanks fitted with level transmitters, rotameters. Process signals from the four tank level transmitters are interfaced with computer. Control algorithm running on the computer sends output to the individual pump variable frequency drive through interfacing units. Tank 1 and Tank 2 are mounted below the other two tanks for receiving water flow by gravity. Each tank outlet opening is fitted with a valve. The connected valve between tank 1 and tank 2 combines water flow path of tank 1 and tank 2. Both pumps 1 and 2 suction from the supply tank. Each pump is fitted with an air cushioned buffer tank to dampen the flow fluctuations from the metering pumps. Discharge from pump 1 is split between tank 1 and tank 3 and flows are indicated be rotameters 1 and 3. Similarly, pump 2 splits its discharge between tank 2 and tank 4 and the split flows are indicated by rotameters 2 and 4. Split of flow from pump 1 and pump 2 can be varied by manual adjustment of valves S_1 and S_2. Tank 1 and Tank 2 also receive gravity flow from tank 3 and tank 4, respectively. A connected valve V5 in fig. 1, it combines the water flow path of tank1 with tank2. Opening of these valves (V1, V2, V3, V4 and V5), and the flow split valves (S_1 and S_2) can be manually adjusted to substantially alter the characteristics of the system. When the connected valve ratio βx take the value over 0, it will create the interacting channel between water process in tank1 and tank2. By the interacting structure, we can assess the performance of control system design in the interacting condition.

What makes the process more complicated is the dependence of split valve opening. The split fraction of flow from a pump going to the lower tank decreases with increase in pump flow. This is a strong source of non-linearity and the process initially starting in minimum phase can transit to non-minimum phase during operation. The Quadruple tank process has two transmission zeros. The position of one of these zeros depends on split fraction γ_1 and γ_2 in valves 1 and 2 respectively. The minimum and non-minimum phase mode can be achieved as

Minimum Phase: $1 < (\gamma_1 + \gamma_2) < 2$

Non-minimum Phase: $0 < (\gamma_1 + \gamma_2) < 1$

III. MODEL DEVELOPMENT

The control objective of modified quadruple tank system is to control water level in two lower tanks i.e. tank 1 and tank 2.

The process has two inputs – flow from pump 1 and pump 2. These are set by signal inputs u_1 and u_2, which are the % output from the controller. There are four levels (h_1, h_2, h_3 and h_4) that are measured, transmitted and are available on-line for the control algorithm to make use of it.

Mass balance and Bernoulli's law yield non-linear plant equation as following [5][6].

$$\frac{dh_1(t)}{dt} = \frac{\gamma_1 k_{p1}}{A}u_1(t) + \frac{\beta_3 a_3}{A}\sqrt{2gh_3(t)} - \frac{\beta_x a_x}{A}\mathrm{sgn}(h_1(t)-h_2(t))\sqrt{2g|h_1(t)-h_2(t)|} - \frac{\beta_1 a_1}{A}\sqrt{2gh_1(t)} \quad (1)$$

$$\frac{dh_2(t)}{dt} = \frac{\gamma_2 k_{p2}}{A}u_2(t) + \frac{\beta_4 a_4}{A}\sqrt{2gh_4(t)} + \frac{\beta_x a_x}{A}\mathrm{sgn}(h_1(t)-h_2(t))\sqrt{2g|h_1(t)-h_2(t)|} - \frac{\beta_2 a_2}{A}\sqrt{2gh_2(t)} \quad (2)$$

$$\frac{dh_3(t)}{dt} = \frac{(1-\gamma_1)k_{p1}}{A}u_1(t) - \frac{\beta_3 a_3}{A}\sqrt{2gh_3(t)} \quad (3)$$

$$\frac{dh_4(t)}{dt} = \frac{(1-\gamma_2)k_{p2}}{A}u_2(t) - \frac{\beta_4 a_4}{A}\sqrt{2gh_4(t)} \quad (4)$$

Where

A : Cross section area of tank= $30cm^2$
a_i : Cross section area of the outlet hole (cm^2)

a_x: Cross section area of the connection hole between tank1 and tank2

$h_i(t)$: water level (cm)

$u_j(t)$: Voltage input of pump (volt)
β_i : Outlet valve ratio
β_x : connected valve ratio between tank 1 and tank2
γ_j : Inlet valve ratio

For minimum phase: $\gamma_1 = 0.70$ & $\gamma_2 = 0.60$

For Non-minimum phase: $\gamma_1 = \gamma_2 = 0.3$

k_{pi} : Gain of pump (cm^3/volt/sec)

g : Specific gravity = 981 cm/s^2

The pump generate a flow proportional to the applied voltage:

$q_{pump,j}(t) = k_{pi} \cdot u_j(t)$, [where j=1,2 & i =1,2]

The flow that split up by the valves to tank 1 is $\gamma_1 k_{p1} u_1(t)$, tank 2 is dynamics. $\gamma_2 k_{p2} u_2(t)$, tank 3 is $(1-\gamma_1) k_{p1} u_1(t)$, tank 4 is $(1-\gamma_2) k_{p2} u_2(t)$. Each of the valves (V1, V2, V3, V4, V5, S1 and S2) has non-linear characteristics and they interact to increase the order of dynamics.

IV. TRANSFER FUNCTION MATRIX

The Laplace transform of linearised model yields to the transfer matrix of the four tank system.

$G(s) = C(SI - A)^{-1} B$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = G(s).\begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

Where, $$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \quad (5)$$

Solution of transfer function matrix is,

$$g_{11}(s)=\frac{\left[\frac{\gamma_1 T_1 k_{p1}}{A}+\frac{(1-\gamma_1)T_1 T_2 k_{p1}/T_x A}{(T_4 s+1)\left(T_2 s+1+(T_2/T_x)\right)}\right]}{\left[\left[T_1 s+1+\frac{T_1}{T_x}\right]-\left[\frac{T_1 T_2/T_x^2}{\left(T_2 s+1+\left(\frac{T_2}{T_x}\right)\right)}\right]\right]}$$

(6)

$$g_{12}(s)=\frac{\left[\frac{\gamma_2 T_2 k_{p2}}{A}+\left(T_2 s+1+\frac{T_2}{T_x}\right).T_x.\frac{(1-\gamma_2).k_{p2}/A}{(T_3 s+1)}\right]}{\left[\frac{T_x}{T_1}.\left(T_2 s+1+\frac{T_2}{T_x}\right).\left(T_1 s+1+\frac{T_1}{T_x}\right)-\frac{T_2}{T_x}\right]}$$

(7)

$g_{21}(s)=$

$$\frac{\left[\frac{\gamma_1 T_1 k_{p1}}{A}+\left(T_1 s+1+\frac{T_1}{T_x}\right).\frac{T_x}{T_2}.\left(\frac{(1-\gamma_1).T_2.k_{p1}/A}{(T_4 s+1)}\right)\right]}{\left[\frac{T_x}{T_2}.\left(T_1 s+1+\frac{T_1}{T_x}\right).\left(T_2 s+1+\frac{T_2}{T_x}\right)-\frac{T_1}{T_x}\right]} \quad (8)$$

$$g_{22}(s)=\frac{\left[\frac{\gamma_2 T_2 k_{p2}}{A}+\frac{(1-\gamma_2)T_1 T_2 k_{p2}/T_x A}{(T_3 s+1)\left(T_1 s+1+\frac{T_1}{T_x}\right)}\right]}{\left[\left[T_2 s+1+\frac{T_2}{T_x}\right]-\left[\frac{T_1 T_2/T_x^2}{\left(T_1 s+1+\frac{T_1}{T_x}\right)}\right]\right]}$$

(9)

Where the time constant:

$$\frac{1}{T_i}=\frac{\beta_i a_i}{A}\sqrt{\frac{g}{2\bar{h}_i}},\ \ i=1,\ldots,4$$

$$\frac{1}{T_x}=\frac{\beta_x a_x}{A}\sqrt{\frac{g}{2\left|\bar{h}_1-\bar{h}_2\right|}}$$

V. FURTHER ANALYSIS OF THE SYSTEM

A. Relative Gain Array (RGA)

For 2x2system, the relative gain array shows how inputs and outputs of the system are coupled. It has the following form. With

$$m=\frac{g_{11}(0).g_{22}(0)}{g_{11}(0).g_{22}(0)-g_{12}(0).g_{21}(0)}$$

(10)

With equation (10) *m* can be found as:

$$m=\frac{(T_x\gamma_2+T_1).(T_x\gamma_1+T_2)}{(T_x+T_1+T_2).(\gamma_1+\gamma_2-1).T_x}$$

(11)

For big value of m, the dominating elements of the transfer matrix are the diagonal elements. Output 1 is effected mostly by input1, output2 by input2. If m is small, output 1 depends mainly on input2, and output 2 on input 1. To know which input mainly affects which output is important for design of PI controllers for the MIMO system.

B. Zeros of the system (Minimum and Non-minimum phase)

The zeros of the transfer matrix equation (5) are defined as the zeros of det[G(s)]. G(s) has four zeros. To find them we have to find the zeros from the solutions of numerator of G(s) is equal zero. From the fig.2 explain the relation between the zeros position on s-plane and valve ratio value γ_1, γ_2 that take the value of ($\gamma_1+\gamma_2$) in (0,2).

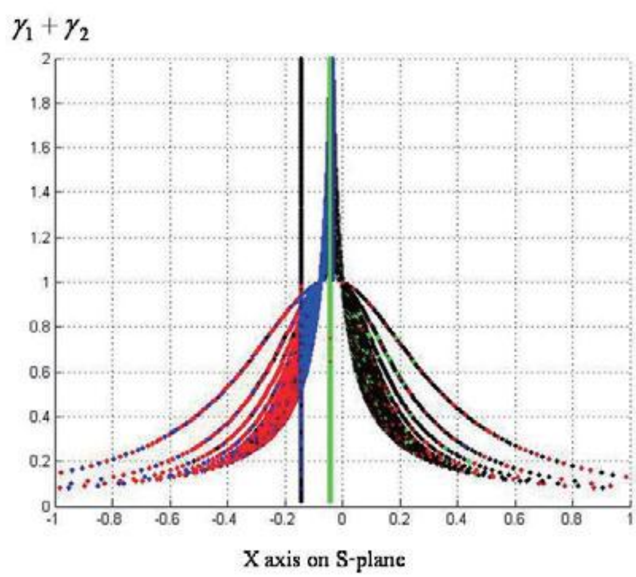

Fig.2. Zeros of G(s) related with flow ratio 0 < ($\gamma_1+\gamma_2$) < 2

The zeros of G(s) can be located either in the left or in the right half-plane. The system is minimum phase (both zeros are in the left half-plane) for 1 < ($\gamma_1+\gamma_2$) < 2 and the system is non-minimum phase for 0 < ($\gamma_1+\gamma_2$) < 1.We found that the zero at origin occur when ($\gamma_1+\gamma_2$) = 1.

VI. STRUCTURE OF PI CONTROLLER DESIGN

In case of minimum phase system, transfer function g_{11} and g_{22} are used for design the controller, but the case of non-minimum phase system the transfer function g_{12} and g_{22} are instead used. The structure of MIMO control system using PI controller for minimum phase and non-minimum phase are shown in Fig. 3 and Fig. 4 respectively. The values of proportional and integrator are found to be

$G_{c1}(s) = \frac{K_{i1}}{s}$, $G_{k1} = K_{p1}$ and $G_{c2}(s) = \frac{K_{i2}}{s}$, $G_{k2} = K_{p2}$

Where

For Minimum Phase

$K_{p1} = K_{p2} = 15$ $\quad K_{i1} = K_{i2} = 5$

For Non-minimum Phase

$K_{p1} = 0.45$, $K_{p2} = 0.44$

$K_{i1} = 0.005$, $K_{i2} = 0.004$

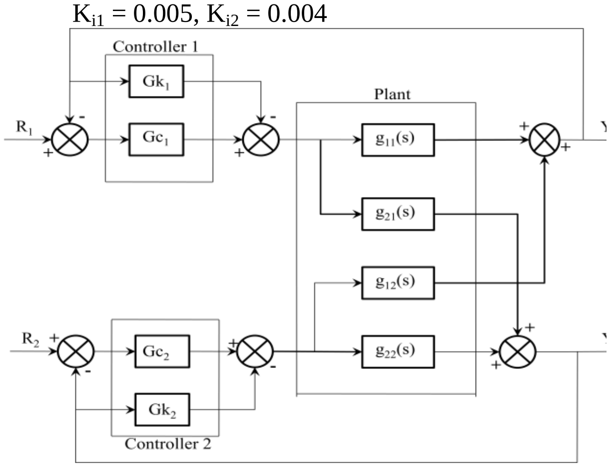

Fig.3 Structure of the MIMO minimum phase control System

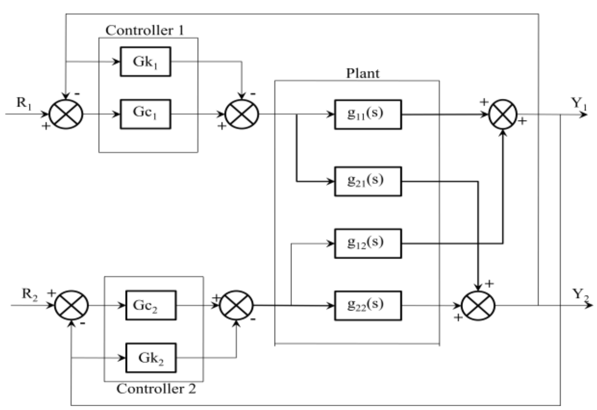

Fig. 4 Structure of the MIMO non-minimum phase control system

VII. SIMULATION RESULTS

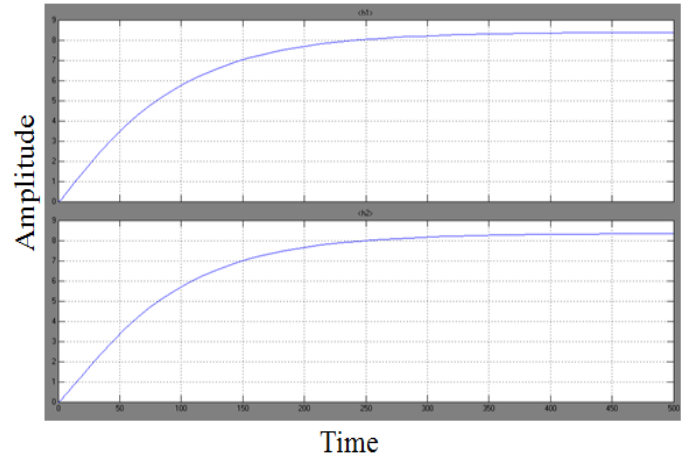

Fig.5 Simulation result of the transfer function matrix for minimum phase

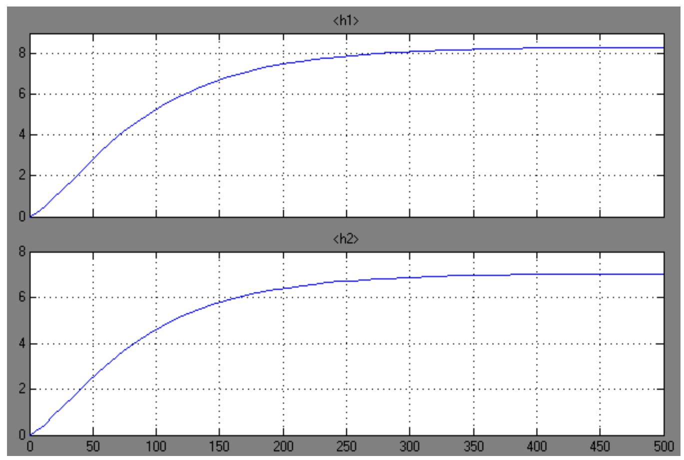

Fig.6 Simulation result of the transfer function matrix for non-minimum phase

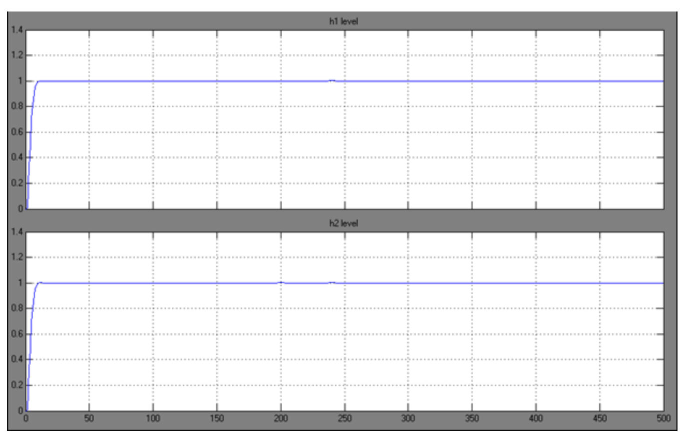

Fig.7 Simulation result of the Modified Quadruple Tank System control for minimum phase using PI Controller

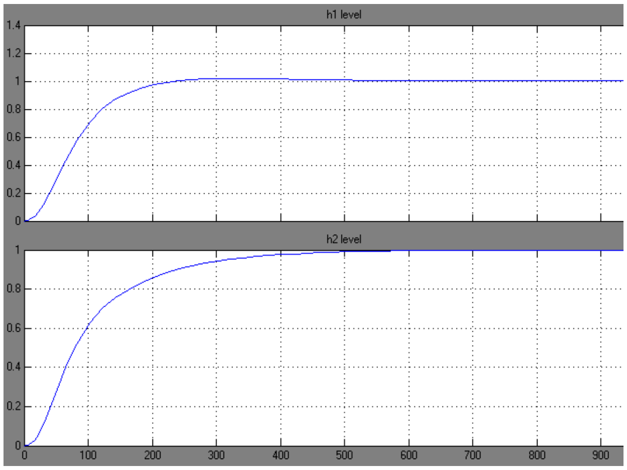

Fig.8 Simulation result of the Modified Quadruple Tank System control for non-minimum phase using PI Controller

VIII. CONCLUSION

From the transfer function simulation results we conclude that The Modified Quadruple tank process system is stable. Hence the process validation is done. The step response of open loop system and PI controller are compared for both minimum phase and non-minimum phase system. And it is improved by using PI controller. Even if the plant is a non-minimum phase system, the PI controller be suitably applied for both tank1 and tank2 and the settling time are satisfied for both tanks.

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