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A Detailed Comparative Study between Reduced Order Luenberger and Reduced Order Das & Ghosal Observer and their Applications

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Abstract:
In this paper a detailed comparative study has been carried out between two well known methods of reduced order observer construction, namely - Reduced Order Luenberger method (1964, 1971) [1, 2] and Reduced Order Das and Ghosal method (1981) [5]. Through proper examples and illustrations, similarities and dissimilarities between the above mentioned methodsand their advantage & disadvantages are explained in this paper. Brief mathematical preliminaries, a few theorems, lemmas and governing equations are also included in this paper according to the context.

***Keywords:* Luenberger Observer, Generalized Matrix Inverse, State Feedback Control, Ackermann's formula, Full & Reduced order observer, Inverted Pendulum.**

1. Introduction:

Till date various observer design procedures have been proposed by several authors for different systems like linear/non linear systems, time invariant / time varying systems, continuous / discrete time systems etc. For linear systems, both full-order and reduced-order observer construction methods [1, 2] were published by D. G. Luenberger (1964, 1971). The methods proposed by Luenberger are very simple and easy to implement. In full order observer all the system states are estimated while in case of reduced order observer only the immeasurable (i.e. states not available for direct measurement) states are estimated. The state estimation becomes essential when state feedback control is applied. Luenberger observer follows some constraint equations and it presumes the structure of the underlying plant for which the observer is to be designed. In contrast Das and Ghosal (1981) observer [5] does not need to satisfy those constraints and also, it does not presume the structure of the plant. It uses generalized matrix inverse technique [4, 5, and 6] for computing the state estimation.

In this paper first both of the methods are discussed briefly and then exhaustive comparison is done on the basis of structure of the observers and performance of them. Finally three numerical examples and simulation results are given for justifying the comparison.

List of Notations and Symbols:

Table 1.1

Serial No	Notation / Symbols	Purpose	Dimension
1.	A	System matrix	n×n
2.	B	Input matrix	n×p
3.	C	Output Matrix	m×n
4.	D	Transmission matrix	m*p
5.	L	This is a special matrix needed in Das and Ghosal Observer design, chosen such that $LL^g = I - C^g C$ is satisfied.	n× (n-m)
6.	L^g	Generalized Inverse of L matrix	(n-m)×m
7.	C^g	Generalized Inverse of C matrix	n×m
8.	A_{11}	Upper left partition of A matrix	m×m
9.	A_{12}	Upper right partition of A matrix	m×(n-m)
10.	A_{21}	Lower left partition of A matrix	(n-m)×m
11.	A_{22}	Lower right partition of A matrix	(n-m)×(n-m)
12.	B_1	Upper partition of B matrix	m×p
13.	B_2	Lower partition of B matrix	(n-m)×p
14.	M	Observer Gain matrix	(n-m)×m
15.	x	State Vector	n×1

16.	$\hat{x}$	Estimated State Vector	(n-m)×1
17.	h	Immeasurable State Vector used in [5]	(n-m)×1
18.	$\hat{q}$	Observer State Vector used in [5]	(n-m)×1
19.	F	General Observer System matrix in the sense of Luenberger	(n-m)×(n-m)
20.	G	General Observer Output matrix in the sense of Luenberger	(n-m)×m
21.	$\bar{B}$	General Observer Input matrix in the sense of Luenberger	(n-m)×p
22.	$\hat{w}$	Observer State Vector used in [1]	(n-m)×1
23.	$\hat{z}$	Observer State Vector used in [1]	(n-m)×1
24.	I_n	Identity matrix	n×n
25.	y	Output Vector	m×1
26.	$\hat{y}$	Observer Output Vector	(n-m) ×1
27.	T	Transformation matrix	(n-m)×n
28.	V	Transformation matrix	n×m
29.	P	Transformation matrix	n×m
30.	$\hat{C}$	Maps $\hat{z}$ with $\hat{x}$	n× (n-m)
31.	$\hat{D}$	Maps y with $\hat{x}$	n×m
32.	u	Control Input	p×1
33.	K	State Feedback Gain matrix	p×n

2. Mathematical Preliminaries:

Incase of Luenberger's method no special mathematical tools were used but Das and Ghosal had used the concept of generalized matrix inverse [7] [8] to derive the dynamics of observer system. Generalized matrix inverse theory is now discussed below in brief:

An equation $Ax = y \dots \dots (1)$ has been taken where A is a given (m×n) matrix, y is a given (m×1) vector, x is an unknown (n×1) vector. Also an (n×m) matrix A^g is taken such that

- $AA^g = (AA^g)^T \dots \dots (2)$
- $A^gA = (A^gA)^T \dots \dots (3)$
- $AA^gA = A \dots \dots (4)$
- $A^gAA^g = A^g \dots \dots (5)$ are satisfied where superscript T indicates transpose.

The $\boldsymbol{A^g}$ is called the **Moore-Penrose** Generalized Matrix Inverse of A and $\boldsymbol{A^g}$ is unique for A. Now eqn. (1) is consistent if and only if $AA^gy = y \dots \dots (6)$ and if eqn. (1) is consistent then its general solution is given by $x = A^g\boldsymbol{y} + (I - A^gA)\boldsymbol{v} \dots \dots (7)$ where I is the identity matrix of proper dimension and $\boldsymbol{v}$ is an arbitrary (n×1) vector (Graybill 1969, Grayville 1975).

Lemma used by das and Ghosal: For an (m×n) matrix C and an (n×k) matrix L, if the linear space spanned by columns of L is equal to the linear space spanned by the columns of $(I - C^gC)$, then L^gL is equal to $(I - C^gC)$. So by this Lemma, proposed by Das and Ghosal, 1981: $\boldsymbol{LL^g = I - C^gC} \dots \dots (8)$.

3. Brief Theory on Observer:

To implement state feedback control [control law is given by $u = r - \boldsymbol{Kx} \dots \dots (9)$] by pole placement, all the state variables are required to be feedback. However, in many practical situations, all the states are not accessible for direct measurement and control purposes; only inputs and outputs can be used to drive a device whose outputs will approximate the state vector. This device (or computer program) is called **State Observer.** Intuitively the observer should have the similar state equations as the original system (i.e. plant) and design criterion should be to minimize the difference between the system output $y = C\boldsymbol{x}$ and the output $\hat{y} = C\hat{\boldsymbol{x}}$ as constructed by the observed state vector $\hat{\boldsymbol{x}}$. This is equivalent to minimization of $\boldsymbol{x} - \hat{\boldsymbol{x}}$**.** Since $\boldsymbol{x}$ is inaccessible, $y - \hat{y}$ is tried to be minimized. The difference $(y - \hat{y})$ is multiplied by a gain matrix (denoted by **M**) of proper dimension and feedback to the input of the observer.

3.1 Reduced Order Luenberger Observer Design:

Any LTI system is described in state space form as $\dot{x} = Ax + Bu \; And \; y = Cx \dots \dots (10)$ (eqn. 1.1 of [1]). Reduced order Luenberger observer is governed by the following equations and conditions.

$TA - FT = GC \dots \dots (11a)$ (Page-600, eqn. 5.5a of [1]; Luenberger Constraint)
In general Luenberger observer dynamics is given by the equation:
$\dot{z}(t) = F\,z(t) + G\,y(t) + \bar{B}\,u(t) \dots \dots (11b)$ (eqn. 2.3 of [1])

$$\hat{x} = \begin{bmatrix} C(t) \\ T(t) \end{bmatrix}^{-1} \begin{bmatrix} y(t) \\ w(t) \end{bmatrix} \dots \dots (12) \quad \text{(eqn. 3.1 of [1])}$$

Entire State vector has been partitioned as $x = \begin{bmatrix} y \\ \dots \\ w \end{bmatrix} \dots \dots (13)$ (page 598 of [1]) and accordingly the system has also been transformed into a partitioned form: $\dot{x} = \begin{bmatrix} A_{11} & \vdots & A_{12} \\ \cdots & \vdots & \cdots \\ A_{21} & \vdots & A_{22} \end{bmatrix} x + \begin{bmatrix} B_1 \\ \cdots \\ B_2 \end{bmatrix} u \dots\dots (14)$

And $y = [I_m \quad \vdots \quad 0]\, x \dots\dots (15)$ If C matrix is not in this form then a coordinate transformation is necessary (page 598 of [1]).

$\dot{w(t)} = A_{22}\, w(t) + A_{21}\, y(t) + B_2 u(t) \dots\dots (16)$ (eqn. 3.3b of [1])

$\dot{y} = A_{12} w + A_{11}\, y + B_1 u \dots\dots (17)$ (eqn. 3.3a of [1])

$\dot{\widehat{w}} = (A_{22} - MA_{12})\widehat{w} + (A_{21} - MA_{11})y + (B_2 - MB_1)u + M\dot{y} \dots\dots (18)$ (eqn. 3.4 of [1])

$\dot{\hat{z}} = (A_{22} - MA_{12})\hat{z} + \{(A_{21} - MA_{11}) + (A_{22} - MA_{12})M\}y + (B_2 - MB_1)u \dots\dots (19)$ (eqn. 3.5 of [1])

$where\ \hat{z} = \widehat{w} - My \dots\dots (20)$ (eqn. 3.6 of [1])

$And\ \ \hat{x} = \hat{C}\hat{z} + \widehat{D}y \dots\dots (21)$

$$where\ \hat{C} = \begin{bmatrix} 0_{m*(n-m)} \\ \cdots \\ I_{(n-m)} \end{bmatrix}\ and\ \widehat{D} = \begin{bmatrix} I_m \\ \cdots \\ M_{(n-m)*m} \end{bmatrix}$$

3.2 Reduced Order Das and Ghosal Observer Design:

Reduced order Das and Ghosal observer is governed by the following equations and conditions.

$x = C^g y + L\, h \dots\dots\dots (22\,)$ (eqn. 13 of [5])

$\dot{h(t)} = L^g AL\, h(t) + L^g AC^g\, y(t) + L^g\, B\, u(t) \dots\dots\dots (23)$ (eqn. 15 of [5])

$\dot{y} = CALh + CAC_g\, y + CB\, u \dots\dots (24)$ (eqn. 18 of [5])

$\dot{\hat{h}} = (L^g AL - MCAL)\hat{h} + (L^g AC^g - MCAC^g)y + (L^g - MCB)u + M\dot{y} \dots\dots (25)$ (eqn. 19 of [5])

$\dot{\hat{q}} = (L^g AL - MCAL)\hat{q} + \{(L^g AC^g - MCAC^g) + (L^g AL - MCAL)M\}y + (L^g - MCB)u \dots\dots (26)$ (eqn. 20 of [5])

$where\ \hat{q} = \hat{h} - My \dots\dots (27)$ (Page-374 of [5])

$And\ \ \hat{x} = L\hat{q} + (C^g + LM)y \dots\dots (28)$ (eqn. 21 of [5])

4. Numerical Examples:

As an example both of the observers have been implemented to estimate the immeasurable states of an inverted pendulum on a moving cart [11]. The system is governed by the state-space representation:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\dfrac{(mL)^2 g}{MmL^2 + J(M+m)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{(M+m)mgL}{MmL^2 + J(M+m)} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{J + (mL^2)}{MmL^2 + J(M+m)} \\ 0 \\ -\dfrac{mL}{MmL^2 + J(M+m)} \end{bmatrix} u \dots\dots (29)$$

$$And\quad y = [1 \quad 0 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \dots\dots (30)$$

$where\ x_1 = z(t), horizontal\ displacement\ of\ the\ cart;$
$x_2 = \dot{z}(t), linear\ velocity\ of\ the\ cart;$
$x_3 = \theta(t), angular\ displacement\ of\ the\ pendulum;$
$x_4 = \dot{\theta}(t), angular\ velocity;$

Putting the numerical values **M=1kg, m=0.15kg, g=9.81m/sec^2, L=1m** we get the equations:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -0.5809 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 4.4537 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.9211 \\ 0 \\ -0.3947 \end{bmatrix} u \dots\dots (31)$$

Luenberger observer has been implemented by following the equations **(19&21):**

$$\begin{bmatrix} \dot{\hat{z}_1} \\ \dot{\hat{z}_2} \\ \dot{\hat{z}_3} \end{bmatrix} = \begin{bmatrix} -12.00 & -0.5809 & 0 \\ 131.6125 & 0 & 1 \\ 463.8396 & 4.4537 & 0 \end{bmatrix} \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \\ \hat{z}_3 \end{bmatrix} + \begin{bmatrix} 0.9211 \\ 0 \\ -0.3947 \end{bmatrix} u + \begin{bmatrix} -67.5 \\ 1115.5 \\ 4979.9 \end{bmatrix} y \dots\dots (32)$$

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_1 \\ \hat{x}_1 \\ \hat{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{z}_1 \\ \hat{z}_1 \\ \hat{z}_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 12 \\ -131.6125 \\ -463.8396 \end{bmatrix} y \dots\dots (33)$$

Das and Ghosal observer has been realized by using the equations **(26&28):**

$$\begin{bmatrix}\dot{\hat{q}}_1\\ \dot{\hat{q}}_2\\ \dot{\hat{q}}_3\end{bmatrix} = \begin{bmatrix}-12.00 & -0.5809 & 0\\ 131.6125 & 0 & 1\\ 463.8396 & 4.4537 & 0\end{bmatrix}\begin{bmatrix}\hat{q}_1\\ \hat{q}_2\\ \hat{q}_3\end{bmatrix} + \begin{bmatrix}0.9211\\ 0\\ -0.3947\end{bmatrix}u + \begin{bmatrix}-67.5\\ 1115.5\\ 4979.9\end{bmatrix}y \dots\dots (34)$$

$$\begin{bmatrix}\hat{x}_1\\ \hat{x}_1\\ \hat{x}_1\\ \hat{x}_1\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\hat{q}_1\\ \hat{q}_1\\ \hat{q}_1\end{bmatrix} + \begin{bmatrix}1\\ 12\\ -131.6125\\ -463.8396\end{bmatrix}y \dots\dots (35)$$

5. Matlab Simulation Results:

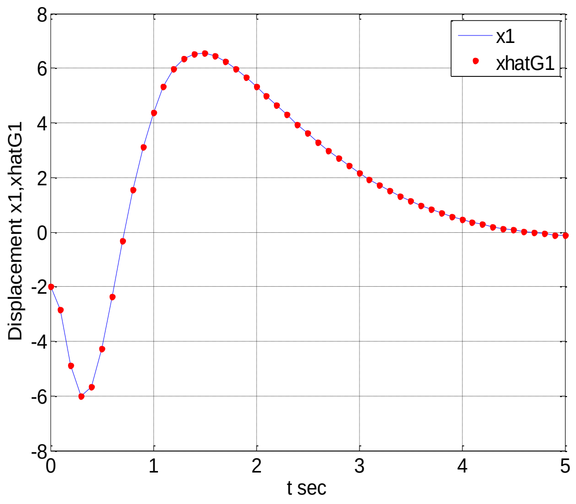

Figure 6.1: **x1 is plant state and xhatG1 is estimated state**

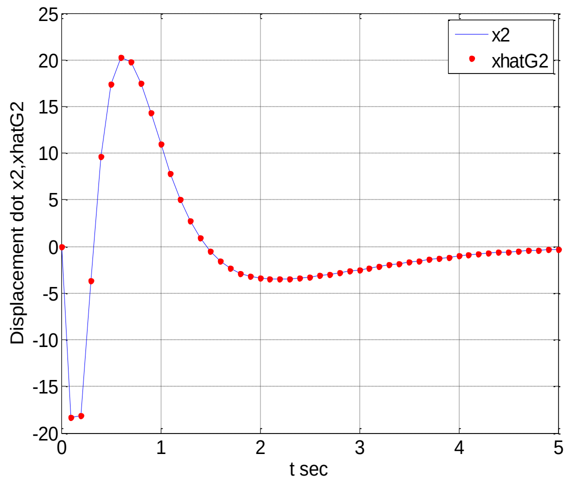

Figure 6.2: **x2 is plant state and xhatG2 is estimated state**

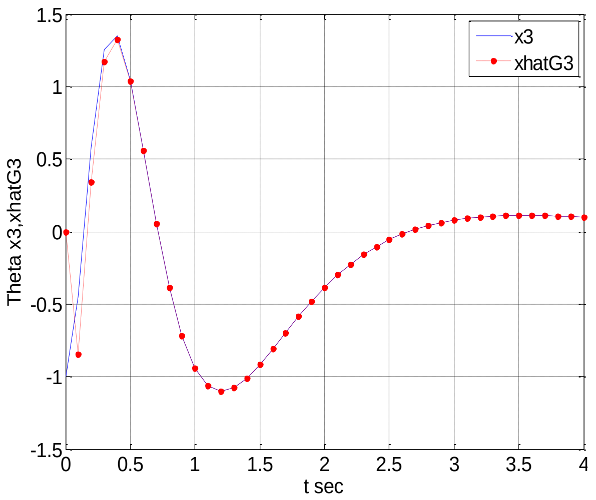

Figure 6.3: **x3 is plant state and xhatG3 is estimated state**

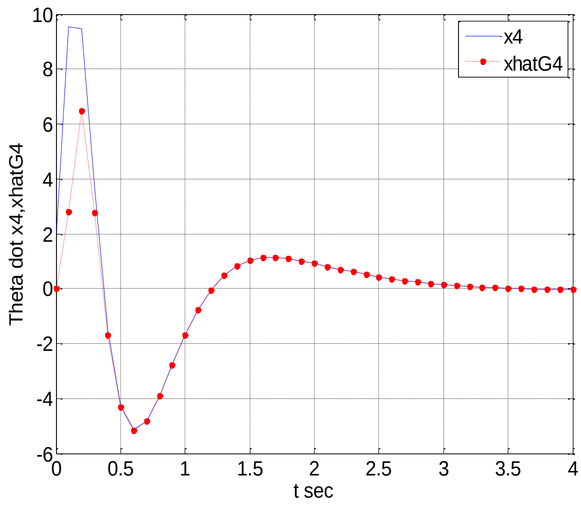

Figure 6.4: **x4 is plant state and xhatG4 is estimated state**

6. Comparative study of Luenberger and Das and Ghosal Observer methods:

A. Structure wise comparison:

6.A.1 Component wise Comparison:

[In the foregoing comparative study
LM →Luenberger's Method and DGM →Das and Ghosal's Method]

I. In LM system state variables can be described as:

$$x = \begin{bmatrix} C(t) \\ T(t) \end{bmatrix}^{-1} \begin{bmatrix} y(t) \\ w(t) \end{bmatrix} \dots\dots (12a)$$

$$Or\ x = [V(t) \quad P(t)] \begin{bmatrix} y(t) \\ w(t) \end{bmatrix} \dots\dots (12b)$$

T, V, P all is transformation matrices of proper dimensions.

In DGM also:

$$x = C^g y + L\,h \;\; eqn. (22)$$

$$Or\ x = [C^g \quad L] \begin{bmatrix} y(t) \\ h(t) \end{bmatrix}$$

II. In LM estimated state variables can be presented as:

$$\hat{x} = \begin{bmatrix} C(t) \\ T_2(t) \end{bmatrix}^{-1} \begin{bmatrix} y(t) \\ z(t) \end{bmatrix} eqn. (12)$$

$$Or\ \hat{x} = [V_2(t) \quad P_2(t)] \begin{bmatrix} y(t) \\ \hat{z}(t) \end{bmatrix}$$

In DGM also:

$$\hat{x} = (C^g + L\,M)\,y + L\,\hat{q} \;\; eqn. (28)$$

$$Or\ \hat{x} = [C^g + LM \quad L] \begin{bmatrix} y \\ \hat{q} \end{bmatrix}$$

III. In LM $\hat{z} = \hat{w} - My$

$$Or\ \hat{z} = [-M \quad I] \begin{bmatrix} y \\ \hat{w} \end{bmatrix}$$

In DGM:

$$\hat{q} = \hat{h} - My$$

$$Or\ \hat{q} = [-M \quad I] \begin{bmatrix} y \\ \hat{h} \end{bmatrix}$$

IV. In LM $\dot{w}(t) = A_{22}\,w(t) + A_{21}\,y(t) + B_2 u(t)\; eqn. (16)$

This can be written in the form:

$$\dot{w}(t) = TAP\,w(t) + TAV\,y(t) + T\,B\,u(t) \dots\dots\dots (16A)$$

Incase of LTI systems (article 2.5, of [7])

In DGM also:

$$\dot{h}(t) = L^g AL\,h(t) + L^g AC^g\,y(t) + L^g B\,u(t)\; eqn. (23)$$

So **structure wise** they are same; both equations contain 3 same types of variables and 3 coefficients.

Comparison on the basis of coefficients:

Table 6.2

Luenberger observer	***Das and Ghosal Observer***	***Dimension wise comparison***	***Product wise comparison***	***Value wise Comparison***
$A_{22} \triangleq TAP$	$L^g AL$	Same	Product of 3 matrices	Same*
$A_{21} \triangleq TAV$	$L^g AC^g$	Same	Product of 3 matrices	Same*
$B_2 \triangleq T\,B$	$L^g B$	Same	Product of 2 matrices	Same*

*this is illustrated in section 5 by showing numeric example.

V. In LM some **constraints** have to be satisfied:

Constraint 1: $F(t)T(t) - T(t)A(t) = \dot{T}(t) - G(t)C(t) \dots\dots (36)$

In DGM this is not required.

Constraint 2:

$$\begin{bmatrix} C(t) \\ T_2(t) \end{bmatrix}^{-1} = [V_2(t) \quad P_2(t)] \dots\dots (37)$$

$$Or\ \begin{bmatrix} C(t) \\ T(t) \end{bmatrix} [V(t) \quad P(t)] = I_n$$

$$Or\ = \begin{bmatrix} I_m & 0_{m\times(n-m)} \\ 0_{(n-m)\times m} & I_{9n-m)} \end{bmatrix} \dots\dots (37a)$$

In DGM this is not required.

VI. For LTI systems constraint 1 becomes $\boldsymbol{TA - FT = GC}\;\; eqn. (11.a)$

So in DGM these are not at all required.

VII. In LM $\boldsymbol{CV = I_m}$

In DGM also $\boldsymbol{CC^g = I_m}$

This is not a constraint for Das and Ghosal Observer. If only C matrix is in the form $[I_m \;\vdots\; 0]$ then only this condition holds. But for Luenberger observer design this relation must have to be satisfied.

VIII. In LM $\boldsymbol{TP = I_{n-m}}$

In DGM also $\boldsymbol{L^g L = I_{n-m}}$

IX. In LM $\boldsymbol{CP = 0_{m\times(n-m)}}$

In DGM also $\boldsymbol{CL = 0_{m\times(n-m)}}$

X. In LM $\boldsymbol{TV} = \mathbf{0}_{(n-m)*m}$

In DGM also $\boldsymbol{L^g C^g} = \mathbf{0}_{(n-m)\times m}$

This is not a constraint for Das and Ghosal Observer. If only C matrix is in the form $[I_m \ \vdots \ 0]$ then only this condition holds. But for Luenberger observer design this relation must have to be satisfied.

XI. In LM the pair $(\boldsymbol{A_{22}}, \boldsymbol{A_{12}})$ has to be completely observable (Lemma 2: Luenberger, 1971) [1].

Similarly in DGM the pair $(\boldsymbol{L^g AL}, \boldsymbol{CAL})$ has to be completely observable.

XII. In LM the output equation is:

$$\dot{y} = A_{12}w + A_{11}\,y + B_1 u \ldots\ldots (17)$$

The above eqn. can be expressed on the form:

$$\dot{y} = CAP\,w + CAV\,y + CB\,u \ldots\ldots (17a)$$

In DGM the output equation is:

$$\dot{y} = CAL\,h + CAC_g\,y + CB\,u \ldots\ldots (24)$$

Comparing eqns. (17, 17a) and (24) we can state that both the equations contain **3 terms;** so **structure wise** they are same.

Now by **comparing the coefficients** we can say that**:**

Table 6.3

Luenberger observer	*Das and Ghosal Observer*	*Dimension wise comparison*	*Product wise comparison*	*Value wise Comparison*
$A_{11} \triangleq CAV$	CAC^g	Same dimension	Product of 3 matrices	Same*
$A_{12} \triangleq CAP$	CAL	Same dimension	Product of 3 matrices	Same*
$B_1 \triangleq CB$	CB	Same dimension	Product of 2 matrices	Same*

*this is illustrated in section 5 by showing numeric example.

XIII. In LM the observer dynamic equation is:

$$\dot{\hat{w}} = (A_{22} - MA_{12})\hat{w} + (A_{21} - MA_{11})y + (B_2 - MB_1)u + M\dot{y} \ldots\ldots (18)$$

In DGM the observer dynamic equation is:

$$\dot{\hat{h}} = (L^g AL - MCAL)\hat{h} + (L^g AC^g - MCAC^g)y + (L^g - MCB)u + M\dot{y} \ldots\ldots (25)$$

Comparing eqns. (18) and (25) we can state that both the equations contain **4 terms**; so **structure wise** they are same. **Coefficients wise** comparison will be done in the next section.

Table 6.4

Luenberger observer	*Das and Ghosal Observer*	*Dimension wise comparison*	*Value wise comparison*
$A_{22} - MA_{12}$	$L^g AL - MCAL$	Same dimension	Same*
$A_{21} - MA_{11}$	$L^g AC^g - MCAC^g$	Same dimension	Same*
$B_2 - MB_1$	$L^g - MCB$	Same dimension	Same*
M	M	Same matrix	Same

*this is illustrated in section 5 by showing numeric example.

XIV. In LM the **T** $(TA - FT = GC)$ matrix must be invertible.

Similarly in DGM the **L** matrix has to be invertible so that $\boldsymbol{L^g}$ exists.

XV. In LM the **T** matrix must have $(\boldsymbol{n-m})$ rows that are linearly independent of the rows of C matrix.

In DGM also $\boldsymbol{L^g}$ matrix must have $(\boldsymbol{n-m})$ rows that are linearly independent of the rows of C matrix.

XVI. If C matrix is in the form $[\boldsymbol{I_m} \ \vdots \ \mathbf{0}]$ then in LM $\hat{\boldsymbol{C}}$ takes the form $\begin{bmatrix} \mathbf{0}_{m*(n-m)} \\ \cdots \\ \boldsymbol{I}_{(n-m)} \end{bmatrix}$

In this specific cases L matrix, used in DGM, is also found to be exactly in the same form i.e. $\begin{bmatrix} \mathbf{0}_{m*(n-m)} \\ \cdots \\ \boldsymbol{I}_{(n-m)} \end{bmatrix}$; Otherwise **L** will be different from $\hat{\boldsymbol{C}}$

XVII. If C matrix is in the form $[\boldsymbol{I_m} \ \vdots \ \mathbf{0}]$ then in LM $\hat{\boldsymbol{D}}$ takes the form $\begin{bmatrix} \boldsymbol{I}_{m*m} \\ \cdots \\ \boldsymbol{M}_{(n-m)*m} \end{bmatrix}$.

Where M is the observer gain matrix generally computed by Ackermann's formula.

In these specific cases, $(\boldsymbol{C^g + LM})$ matrix, used in DGM, is also found to be exactly in the same form i.e. $\begin{bmatrix} \boldsymbol{I}_{m*m} \\ \cdots \\ \boldsymbol{M}_{(n-m)*m} \end{bmatrix}$.

XVIII. In LM observer error dynamics is given by:

$\dot{\boldsymbol{e}}(\boldsymbol{t}) = (\boldsymbol{A_{22}} - \boldsymbol{MA_{12}})\,\boldsymbol{e}(\boldsymbol{t}) \ldots\ldots (38)$ **(**eqn. 2.26, page-33 of [7])

In DGM observer error dynamics is given by:

$$\dot{\boldsymbol{e}}(\boldsymbol{t}) = (\boldsymbol{L_g AL} - \boldsymbol{MCAL})\,\boldsymbol{e}(\boldsymbol{t}) \ldots\ldots (39)$$

XIX. In case of closed loop system using state feedback ($\boldsymbol{u} = -\boldsymbol{K}\hat{\boldsymbol{x}}$); $\boldsymbol{K} \rightarrow$ $state\ feedback\ gain\ matrix$ **K** is bound by the constraint $\boldsymbol{K} = \boldsymbol{ET} + \boldsymbol{DC}$ … … (40) (eqn. 5.5b, page-598 of [1]).

But in DGM there is no such constraint on selection of **K** matrix.

XX. In LM **C** matrix should be in the form $[\boldsymbol{I_m} \ \vdots \ \boldsymbol{0}]$; otherwise (for instance $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$) a coordinate transformation is required:

$\bar{\boldsymbol{x}} = \boldsymbol{Rx}$; $where\ R = \begin{bmatrix} \boldsymbol{C}_{\boldsymbol{m}\times\boldsymbol{n}} \\ \cdots \\ \boldsymbol{N}_{(\boldsymbol{n}-\boldsymbol{m})\times\boldsymbol{n}} \end{bmatrix}$ $is\ the\ coordinate\ transformation\ matrix.$

In DGM no such specific form of **C** matrix is required.

XXI. If there is repeated term or redundancy (*more than one element in a row*) in **C** matrix like $C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ then LM will not work.

But there is no such constraint for redundancy terms present in **C** matrix incase of DGM

XXII. LM presumes that the observer structure is basically the same as that of the plant.

But in DGM observer structure is not presumed. A generalized approach is adapted.

XXIII. Luenberger observer works equally well for zero and non-zero initial conditions of plant or observer or both.

Same facilities are also there in Das and Ghosal observer.

XXIV. Observer poles can be arbitrarily placed (they should lie in the left half of s plane so that $\boldsymbol{A_{22}} - \boldsymbol{MA_{12}}$ is a stability matrix i.e. the observer system has to be asymptotically stable.

Same argument is applicable to DGM: here also $\boldsymbol{L^g AL} - \boldsymbol{MCAL}$ has to be a stability matrix.

XXV. For state feedback control design problem the closed loop system poles (i.e. eigen values of $\boldsymbol{A} - \boldsymbol{BK}$) and observer poles (i.e. eigen values of $\boldsymbol{A_{22}} - \boldsymbol{MA_{12}}$) are completely different and independent of each other.

Same logic is true for DGM also.

6.A.2 Coefficient wise comparison based on final observer dynamic equation:

In LM rewriting the equations (19&21)

$$\dot{\hat{z}} = (A_{22} - MA_{12})\hat{z} + \{(A_{21} - MA_{11}) + (A_{22} - MA_{12})M\}y + (B_2 - MB_1)u \ \ldots\ldots (19)$$

$where\ \hat{z} = \widehat{w} - My$

And $\hat{x} = \hat{C}\hat{q} + \widehat{D}y \ \ldots\ldots (21)$

In DGM rewriting the equations (26&28)

$$\dot{\hat{q}} = (L^g AL - MCAL)\hat{q} + \{(L^g AC^g - MCAC^g) + (L^g AL - MCAL)M\}y + (L^g - MCB)u \ \ldots\ldots (26)$$

$where\ \hat{q} = \hat{h} - My$

And $\hat{x} = L\hat{q} + (C^g + LM)y \ \ldots\ldots (28)$

Now comparing eqns. (19&21) with (26&28) we get the following observations presented in table 6.5

B. Performance wise comparison:

It has been seen from Matlab simulation results that the performance of both the observers are exactly same. This can be justified by comparing the eqns. (32) & (34) and (33) & (35) of the inverted pendulum system taken in this paper. Numerically they are exactly same and similar.

Table 6.5

In case of Luenberger observer	*In case of Das and Ghosal Observer*	*Dimension wise comparison*	*Value wise comparison*
A_{22}	$L_g AL$	(n-m)×(n-m) both;	Same*
A_{12}	CAL	m×(n-m) both;	Same*
A_{21}	$L^g AC^g$	(n-m)×m both;	Same*
A_{11}	CAC^g	m×m both;	Same*
B_2	L^g	(n-m)×p both;	Same*
B_1	CB	m×p both;	Same*
M	M	(n-m)×m	Same*
$\hat{C}$	L	n×(n-m) both;	Same*
$\widehat{D}$	$C^g + LM$	n×m both;	Same*

***this is illustrated in section 5 by showing numeric example.

7. Future Scope of work:

Till now reduced order Das and Ghosal observer construction method has been implemented for linear time invariant (LTI) systems. It can be extended for linear time varying systems, non linear systems and lti systems with measurable disturbance inputs. Since Das and Ghosal observer does not presume the plant structure so it will be of great importance if applied to physical systems, which are mostly non linear.

8. Conclusion:

In this paper a detailed and exhausted comparative study between reduced order Luenberger and reduced order Das and Ghosal observer construction procedure has been done. It has come out from the study that basically these two types of observers are almost same as structure and performance are concerned. There are some advantages of Das and Ghosal's method over Luenberger's method (indicated by comparison points: XX, XXI, XXII). Besides Luenberger's method has to obey some constraints while Das and Ghosal's method does not need to obey such constraints. Finally, a practical numerical example has been chosen (Inverted Pendulum on a moving cart) and both the observers have been applied to the system to test their response and performance.

9. Reference and Bibliography:

[1] D.G. Luenberger, "An Introduction to Observers", IEEE Transactions on Automatic Control, vol. AC-16, no. 6, pp. 596-602, December. 1971.
[2] D.G. Luenberger, "Observing the states of a linear system", IEEE Transactions Mil. Electron. vol. MIL-8, pp. 74-80, April. 1964.
[3] D.G. Luenberger, "Observers for Multivariable systems", IEEE Transactions on Automatic Control, vol. AC-11, pp. 190-197, April. 1966.
[4] D.G. Luenberger, "Canonical forms for linear multivariable systems", IEEE Transactions on Automatic Control (Short Papers), vol. AC-12, pp. 290-293, June. 1967
[5] G. Das and T.K. Ghosal, "Reduced-order observer construction by generalized matrix inverse", International Journal of Control, vol. 33, no. 2, pp. 371-378, 1981.
[6] Victor Lovass-Nagy, Richard J. Miller and David L. Powers, "An Introduction to the Application of the Simplest Matrix-Generalized Inverse in System Science", IEEE Transactions on Circuits and Systems), vol. CAS-25, no. 9, pp. 766-771, Sep. 1978
[7] F.A. Graybill, "Introduction to Matrices with Applications in Statistics", Belmont, CA: Wadsworth, 1969.
[8] A. Ben-Israel and T.N.E. Greville, "Generalized Inverses, Theory and Applications", New York: Wiley, 1974
[9] J. O'Reilly, "Observers for Linear Systems", London, Academic Press, 1983.
[10] K. Ogata, "Modern Control Engineering", 5th Edition, New Delhi, Prentice Hall of India Pvt. Ltd, 2010
[11] M. Gopal, "Modern Control System Theory", 2nd Edition, New Delhi, Wiley Eastern Limited, April, 1993.
[12] I.J. Nagrath and M. Gopal, "Control Systems Engineering", 4th Edition, New Delhi, New Age International Pvt. Ltd. Publishers, 2005
[13] Elbert Hendrics, Ole Jannerup and Paul Hasse Sorensen, "Linear Systems Control – Deterministic and stochastic Methods", 1st Edition, Berlin, Springer Publishers, 2008
[14] R. K. Bansal, A. K. Goel and M. K. Sharma, "Matlab and its Applications in Engineering, Version 7.5", 2nd Edition, Noida, Pearson Education, 2010
[15] Ajit Kumar Mandal, "Introduction to Control Engineering – Modeling, Analysis and Design", 1st Edition, New Delhi, New Age International Pvt. Ltd. Publishers, 2006